WEEKLY TEST TYM TEST - 21 Balliwala
SOLUTION Date 15-09-2019

[PHYSICS]

1. The co-ordinates of 3 vertices are given by:

$A:(1,3),\quad B:(2,-4),\quad C:(x_3,y_3)$

We know that for a triangular plate the centre of mass lies at the centroid of the triangle.

$$\therefore (X_{CM}, Y_{CM}) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$
$$= \left(\frac{1+2+x_3}{3}, \frac{3-4+y_3}{3}\right)$$

But it is given that;

$$(X_{CM}, Y_{CM}) = (0, 0)$$

$\therefore \quad \frac{3+x_3}{3} = 0 \quad \text{or} \quad x_3 = -3$

and $\quad \frac{-1+y_3}{3} = 0 \quad \text{or} \quad y_3 = 1$

Hence, $\quad (x_3, y_3) = (-3, 1)$.

2. The co-ordinates of the corners of the square are (0, 0), (2, 0), (2, 2), (0, 2). Hence,

$$X_{CM} = \frac{m_1x_1+m_2x_2+m_3x_3+m_4x_4}{m_1+m_2+m_3+m_4}$$
$$= \frac{2\times0+3\times2+5\times2+8\times0}{2+3+5+8} = \frac{16}{18} = \frac{8}{9}\text{ m}$$
$$Y_{CM} = \frac{m_1y_1+m_2y_2+m_3y_3+m_4y_4}{m_1+m_2+m_3+m_4}$$
$$= \frac{2\times0+3\times0+5\times2+8\times2}{2+3+5+8} = \frac{26}{18} = \frac{13}{9}\text{ m}$$

$\therefore$ Co-ordinates of the centre of mass $= \left(\frac{8}{9}, \frac{13}{9}\right)$.

3. Mass of the disc removed $= \frac{M}{\pi R^2} \times \pi \left(\frac{R}{2}\right)^2 = \frac{M}{4}$

Remaining mass $= M - \frac{M}{4} = \frac{3M}{4}$

Let the origin of the co-ordinate system coincide with the centre of mass of whole disc. Now, we know that;

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

X_{CM} will be zero, when

$$m_2 x_2 = -m_1 x_1$$

$\therefore \quad x_2 = -\frac{m_1}{m_2} x_1$

Here, $\quad m_1 = \frac{M}{4}, \quad x_1 = \frac{R}{2}$

and $\quad m_2 = \frac{3M}{4}$ (for remaining mass)

Hence, $\quad x_2 = -\frac{M/4}{3M/4} \cdot \frac{R}{2} = \frac{-R}{6}$

i.e., $\frac{R}{6}$ from the centre (on LHS).

4. Mass of the disc removed $= \frac{M}{\pi (28)^2} \times \pi (21)^2 = \frac{9M}{16}$

Remaining mass $= \frac{7M}{16}$

Using the same method as followed in the above question,

$$\frac{7M}{16} \times OO_2 = \frac{9M}{16} \times OO_1$$

As, $\quad OO_1 = (28 - 21)$ cm $= 7$ cm

Hence, $\quad OO_2 = \frac{9}{7} \times 7$ cm $= 9$ cm .

5. Let the circular disc of radius a be made up of the circular section of radius b and remainder. Further let the line of symmetry joining the centres O and O_1 be the x-axis with O as origin. The centre of mass of the disc of radius a will be given by:

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad \text{...(i)}$$

while Y_{CM} and Z_{CM} will be zero (as for all points on x-axis, y and $z = 0$)

If σ be the density of the material of disc,

$m_1 = \pi b^2 \sigma \quad$ and $\quad x_1 = c$

$m_2 = \pi (a^2 - b^2)\sigma \quad$ and $\quad x_2 = ?$

$M = (m_1 + m_2) = \pi a^2 \sigma \quad$ and $\quad X_{CM} = 0$

From eqn. (i),

$$0 = \frac{\pi b^2 \sigma (c) + \pi (a^2 - b^2)\sigma x_2}{\pi a^2 \sigma}$$

$\therefore \quad x_2 = \frac{-cb^2}{(a^2 - b^2)}$

i.e., the centre of mass of the remainder (say O_2) is at a distance $cb^2/(a^2 - b^2)$ to the left of O on the line joining the centres O and O_1.

6. Let ρ be the density of lead.

Then, $M = \frac{4}{3}\pi R^3\rho$ = mass of total sphere

$m_1 = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\rho$ = mass of removed part $= \frac{M}{8}$

$m_2 = M - \frac{M}{8} = \frac{7M}{8}$ = mass of remaining sphere

Choosing the centre of big sphere as the origin,

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

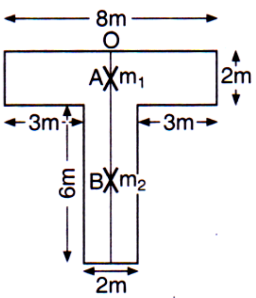
$$0 = \frac{\frac{M}{8}\times\frac{R}{2} + \frac{7M}{8}\times x_2}{M}$$

Solving, we get; $x_2 = \frac{-R}{14}$

i.e., centre of mass of hollowed sphere would be at a distance of $R/14$ on left of O,

i.e., shift in centre of mass $= \frac{R}{14}$.

7. Taking parts A and B as two bodies of same system,



$m_1 = l \times b \times \sigma = 8 \times 2 \times \sigma = 16\sigma$

$m_2 = l \times b \times \sigma = 6 \times 2 \times \sigma = 12\sigma$

Choosing O as the origin,

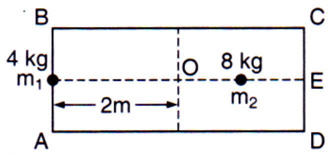
$x_1 = 1$ m, $x_2 = 2 + 3 = 5$ m

$$\therefore \quad X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{16\sigma \times 1 + 12\sigma \times 5}{16\sigma + 12\sigma} = \frac{19}{7}$$

$= 2.7$ m from O.

8. Here, $m_1 = 4$ kg; $x_1 = 2$ m



$m_2 = 8$ kg; $x_2 = ?$

$X_{CM} = 0$

$$\therefore \quad X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$0 = \frac{4\times 2 + 8x_2}{4 + 8}$$

$$\therefore \quad x_2 = -\frac{8}{8} = -1\text{ m}.$$

$\therefore m_2$ (= 8 kg) must be placed at 1 m from O on OE.

9. If the density of cone be ρ, then its mass will be,

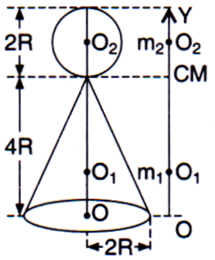
$$m_1 = \frac{1}{3}\pi(2R)^2(4R)\rho = \frac{16}{3}\pi R^3\rho$$

and its centre of mass will be at a height

$$\frac{h}{4} = \frac{4R}{4} = R$$

from O on the line of symmetry,

i.e., $y_1 = R$



Similarly, the mass of the sphere,

$$m_2 = \frac{4}{3}\pi R^3(12\rho) = 16\pi R^3\rho = 3\,m_1$$

and its centre of mass will be at its centre O_2, *i.e.*, $y_2 = 4R + R = 5R$ (from O).

Now, treating the sphere and cone as point masses with their masses concentrated at their centres of mass respectively and taking the line of symmetry as y-axis with origin at O, the centre of mass of the toy is given by:

$$Y_{CM} = \frac{m_1\,y_1 + m_2\,y_2}{m_1 + m_2} = \frac{m_1 \times R + 3m_1 \times 5R}{m_1 + 3m_1} = 4R$$

i.e., centre of mass of the toy is at a distance of $4R$ from O on the line of symmetry, *i.e.*, at the apex of the cone.

10. According to the equation of motion of the centre of mass,

$$M\,\vec{a}_{CM} = \vec{F}_{ext.}$$

If $\vec{F}_{ext.} = 0,\ \vec{a}_{CM} = 0$

i.e., $\vec{v}_{CM} = \text{constant}$

i.e., if no external force acts on a system (or resultant external force acting on a system is zero) the velocity of its centre of mass remains constant (*i.e.*, **velocity of the centre of mass is unaffected by internal forces**). Hence, the kinetic energy and momentum of the system also remain constant.

So, if the centre of mass of a system is at rest (or in the state of uniform motion) it will remain at rest (or in the state of uniform motion) unless acted upon by an external force. Thus, if $\vec{F}_{ext.} = 0$, it is possible that the position of the centre of mass may change at a constant rate.

11. As discussed in the above question 10, if $\vec{F}_{ext.} = 0$, the velocity of the centre of mass of the system remains constant and is not affected by internal forces whatever may be their direction of action.

12. The two bodies will move towards their common centre of mass but the location of the centre of mass will remain unchanged, *i.e.*, CM remains at rest w.r.t. A as well as B.

13. Initially, the particles are at rest, so the velocity of the centre of mass,

$$\vec{v}_{CM} = \frac{m_1 \times 0 + m_2 \times 0}{m_1 + m_2} = 0$$

As here $\vec{F}_{ext.} = 0$, so

$$\vec{v}_{CM} = \text{constant}$$

i.e., $\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = 0$ (at all instants)

or $m_1 \vec{v}_1 + m_2 \vec{v}_2 = 0$

or $m_1 \frac{\Delta\vec{r}_1}{\Delta t} + m_2 \frac{\Delta\vec{r}_2}{\Delta t} = 0$

or $m_1 \Delta\vec{r}_1 + m_2 \Delta\vec{r}_2 = 0$ (as $\Delta t \neq \infty$)

or $m_1\vec{d}_1 + m_2\vec{d}_2 = 0$ (with $\Delta\vec{r} = \vec{d}$)

or $m_1 d_1 - m_2 d_2 = 0$

(as direction of $\vec{d}_2$ is opposite to $\vec{d}_1$)

or $m_1 d_1 = m_2 d_2$

But given that $d_1 + d_2 = d$, so that

$$d_1 = \frac{m_2 d}{m_1 + m_2} \quad \text{and} \quad d_2 = \frac{m_1 d}{m_1 + m_2}$$

Now, as d_1 and d_2 represent the position of the centre of the mass relative to m_1 and m_2 respectively, the particles will collide at the centre of mass of the system.

14. As seen in question 13 two particles will meet at their centre of mass

∴ Distance of the centre of mass from 8 kg mass

$$= \frac{8 \times 0 + 4 \times 12}{8 + 4} = 4 \text{ m}.$$

15.

[CHEMISTR]

16. As it absorbs heat, $q = +208$ J

$$w_{rev} = -2.303 nRT \log_{10}\left(\frac{V_2}{V_1}\right)$$

$$w_{rev} = -2.303 \times (0.04) \times 8.314 \times 310 \ \log_{10}\left(\frac{375}{50}\right)$$

∴ $w_{rev} = -207.76 \approx -208$ J

17. For isothermal reversible expansion of an ideal gas volume V_1 to V_2 the work done is given as :

18 $W = -2.303nRT\log\frac{V_2}{V_1}$

$= -2.303 \times 1 \times 8.314 \times 300 \times \log\frac{20}{10}$

$= -2.303 \times 8.314 \times 300 \times 0.3010 = -1729$ joules

Work done = – 1729 joules

19. Volume depends on the mass of the system.

20.

21. As internal energy is a function of temperature, therefore $\Delta U = 0$

22.

23. For an adiabatic process neither heat enters or leaves the system

$\therefore \quad q = 0$.

24.

ΔE and ΔH both are zero in case of cyclic process. [Also, for isothermal free or reversible expansion of ideal gas, ΔE and ΔH both are zero].

25.

26.

In case of thermodynamic equilibrium $\Delta V, \Delta P, \Delta T$ and Δn all have to be zero.

27.

28.

$$W_{\text{expansion}} = -P\Delta V$$

$$= -(1\times 10^5\,\text{Nm}^{-2})[(1\times 10^{-2} - 1\times 10^{-3})\,\text{m}^3]$$

$$= -10^5 \times (10\times 10^{-3} - 1\times 10^{-3})\,\text{Nm}$$

$$= -10^5 \times 9\times 10^{-3}\,\text{J} = -9\times 10^2\,\text{J} = \mathbf{-900\ J}$$

29.

$$q = \mathbf{+200\ J}$$

$$W = -P\Delta V = -1\times(20-10) = -10\,\text{atm L}$$

$$= -10\times 101.3\,\text{J} = \mathbf{-1013\ J}$$

$$\Delta E = q + W = (200-1013)\,\text{J} = \mathbf{-813\ J}$$

30.

$$\frac{V_2}{V_1} = \frac{1}{10}$$

$$W\text{ (on the system)} = -2.303nRT\log\frac{V_2}{V_1} = -2.303\times 1\times 2\times 500\log\frac{1}{10}\,\text{cal}$$

$$= +\frac{2.303\times 2\times 500}{1000}\,\text{kcal} = \mathbf{+2.303\ kcal}$$